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Augmented Motion Models for Constrained Position Tracking with Kalman Filters

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***Abstract*—Accurate position tracking is a crucial task in many applications ranging from car navigation over robot control to sports analysis. In order to improve the accuracy of position tracking, we introduce a novel method for constraining Kalman filters by incorporating prior knowledge in an augmented motion model. In contrast to previously reported methods, our approach does not require cumbersome tuning of additional filter parameters and causes less computational overhead. We demonstrate our method in the context of sports analysis in athletics. Using 34 data sets recorded during 400 m and 800 m runs, we compare our approach to unconstrained and pseudo-measurement filters. The presented augmented motion model in conjunction with an Extended Kalman Filter (EKF) reduced the root mean square error of the filtered output by 60 % compared to unconstrained filtering and by 50 % compared to a pseudo-measurement EKF.**

I. Introduction

The acquisition of accurate position data is the foundation for a plethora of possible applications. Position data of persons or objects can be used for asset tracking, surveillance of production lines or car and robot navigation. Also in sports monitoring and analysis, position measurement plays an important role. From the position data of athletes, various performance indicators such as momentary and average speeds, covered distances, split and finish times can be obtained in real-time. This information can help to optimize the training success and to avoid injuries.

Different systems for position tracking in sports exist, including Global Positioning Systems (GPS) and Local Positioning Systems (LPS). All of these systems suffer from measurement noise, thus making filtering of the raw measurements necessary. The employed filter needs to meet the following requirements to be used in sports applications. First, filtering has to be conducted in real-time to allow live feedback and broadcasting. Second, the filter should be able to account for constraints, since the position that is to be filtered, often cannot take arbitrary values in reality. For example, a runner in a track and field competition is only allowed to move within the bounds of the defined track, according to the rules. Similar constraints can be found in many other sports (e.g. track cycling, speed skating or in horse and dog races) but also in car or robot navigation where movements are constrained by the geometry of streets and paths.

The widely used Kalman Filter (KF) provides a powerful means for real-time filtering by combining model information about the analyzed process and measurements in a recursive processing procedure. Several extensions of the KF required for the use in non-linear cases exist, including the Extended Kalman Filter (EKF) [1], Unscented Kalman Filter (UKF) [2] and Square-Root Unscented Kalman Filter (SRUKF) [3]. These filters do not naturally handle constraints, but several methods for incorporation of constraints in KFs have been presented in the literature. These constraints can be divided into hard constraints and soft constraints. While hard constraints have to be satisfied exactly, soft constraints only need to be satisfied approximately.

If an object is moving along a known path (e.g. an athlete on a running track or a car on a street), one possibility for improving tracking accuracy of this object would be to constrain the orientation of the object's movement to the orientation of the path. If a hard equality constraint is applied, both orientations must be exactly the same. Thus, one athlete overtaking another or a lane change of a car could not be represented, since this would require a deviation of the movement orientation from the path orientation. In reality, the trajectories of the tracked objects can vary within the width of the path. Therefore, restrictions due to defined paths can be interpreted as soft constraints. This allows a more flexible tracking than using hard constraints. In fact, Simon [4] argues, that most practical systems should be modeled with soft constraints instead of hard constraints. Three main approaches for the incorporation of soft constraints in KFs exist [4]: introduction of pseudo-measurements, projection of the state estimates towards the constraints and regularization.

The incorporation of soft constraints in an EKF by means of pseudo-measurements was described by Alouani and Blair [5] and by Tahk and Speyer [6]. Moreover, Lahrech and Noyer [7] described how map information can be used as pseudo-measurements for car navigation. The idea of introducing pseudo-measurements was adapted for the UKF by

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Teixeira et al. [8]. The introduction of pseudo-measurements requires the additional estimation of the corresponding pseudo-measurement covariance matrix. Moreover, adding pseudo-measurements increases the dimension of the observation vector, thus leading to an increase in computational complexity. Another disadvantage of this approach lies in the fact, that it mixes model information with observed data. This renders common outlier detection methods for Kalman filters ineffective, since they depend on the comparison between predicted and actual measurements.

A projection approach for incorporating soft constraints was presented in a publication by Massicotte [9]. There, a positivity constraint was softly imposed on the state estimate by scaling of the state variables, i.e. by projection toward the constraint surface. The scaling factor for correction needed to be determined empirically. This makes the tuning of this parameter cumbersome and prone to errors. Another projection approach was used in [10]. There, the position of a race car was estimated from GPS measurements by projecting the estimated position towards the track surface in the direction of its normal vector. This led to an improved position accuracy along this axis, but the accuracy of the position on the track surface decreased.

A regularization approach was presented by Simon and Simon [11]. They introduced a penalty term into the Kalman filter to include a priori information. A downside of this scheme is its heuristic nature, which makes it difficult to find the optimal settings for the application at hand. Moreover, computational time for determining the constrained state estimate is increased.

Despite the numerous publications about how to incorporate soft constraints in Kalman-based filters, the available approaches all suffer from one or more of the following shortcomings. First, computational cost is considerably increased, thus limiting the real-time capability of the signal filtering. Second, additional parameters are required. If these parameters have to be determined empirically, this reduces the practicality of these methods and may also lead to suboptimal results. Third, other improvements of Kalman-based filters, such as outlier rejection or optimal smoothing are not compatible with the methods presented in the literature.

The purpose of this paper is to introduce an alternative filtering approach that overcomes these limitations. We argue that the missing agreement between the state estimates and known restrictions is often caused by inappropriate system modeling. Therefore, we propose to include soft constraints directly in an augmented motion model. The proposed approach can easily be implemented and can be combined with different forms of the Kalman filter. Moreover, the computational overhead, in comparison to an unconstrained solution, is small and no additional parameters are required. Our method is applicable for object tracking whenever additional a priori information is available. We demonstrate our method in a sports context.

II. METHODS

A. Fundamentals

All Kalman-based filters are recursive filters. For each discrete time point k, a prediction step and an update step are executed to obtain an estimate of the current state of the analyzed system. The prediction step consists of the calculation of an a priori state estimate $x_{k+1|k}$. The prediction is performed using measurements available up to k and the process model. When a new measurement for time point $k+1$ becomes available, the a posteriori state estimate $x_{k+1|k+1}$ is calculated using the a priori state estimate, the new measurement and the sensor model. This second part is the update step.

Probably the most widely used approach for Kalman-filtering of non-linear systems is the Extended Kalman Filter. In the EKF, non-linear models are approximated using point-wise linearization. This leads to linearization errors, since higher order terms are ignored. Linearization can be avoided by using the UKF instead. In the UKF, the representation of the uncertainty of the state estimate is transformed from a covariance matrix to a set of sigma points. Any non-linear process model can be directly applied to these sigma points without linearization. This method is accurate to the third order for Gaussian systems [12]. However, the UKF suffers from numerical instability, i.e. the positive-definiteness of the state covariance matrix is not guaranteed. To circumvent this problem and to decrease computational cost, the SRUKF was developed by van der Merwe and Wan. A detailed description of the varieties of Kalman filters can be found in [1] and [3].

B. Kalman Filters with Augmented Motion Models

The basic principle of Kalman Filters is to improve the accuracy of the analyzed data by combining the noisy measurements with a model of the underlying process or motion. Hence, the achievable quality of the filtered state estimates depends on the quality of the measurements and on the quality of the motion model. Usually, the quality of the measurements is defined by the available hardware and can not easily be improved. Therefore, the performance of the filter critically depends on the choice of the motion model. When the EKF, UKF or SRUKF are employed, the possible choice of model includes both linear and non-linear variants. Numerous different motion models are available [13] such as the constant velocity model, the constant acceleration model and the constant turn model. In general, these models are simplifications and thus do not encompass all nuances of the real motion of the object of interest.

Consider, for example, the motion of an athlete moving on an oval running track as depicted in Fig. 1. For this case, we chose a state model that comprises the target position in 2D, described by the coordinates p_x and p_y, the norm of the translational speed v, the norm of the acceleration a, the 2D orientation ϕ and the turning rate ω with respect to the current position. The state vector x is given by

$$x = [p_x \; p_y \; v \; a \; \phi \; \omega]^T. \tag{1}$$

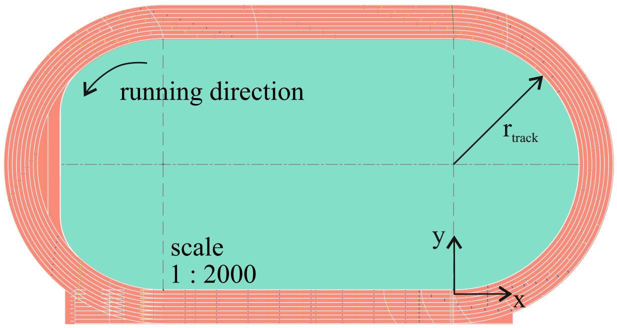

Figure 1. Official running track (modified from [14]).

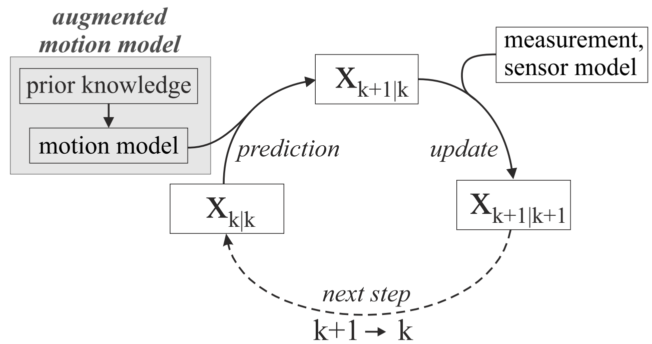

Figure 2. Kalman Filter with augmented motion model for incorporation of prior knowledge

Positioning systems usually provide only observations of the position, hence the linear sensor model can be described in matrix form by

$$C = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 \end{bmatrix}. \tag{2}$$

Since not all state parameters are measured, simplifying assumptions about the unobserved state parameters are necessary. Assuming a circular motion with constant radius and constant acceleration norm leads to the following motion model:

$$\begin{bmatrix} p_{x,k+1} \\ p_{y,k+1} \\ v_{k+1} \\ a_{k+1} \\ \phi_{k+1} \\ \omega_{k+1} \end{bmatrix} = \begin{bmatrix} p_{x,k} + T_k(v_k + a_k T_k)\cos(\phi_k + \omega_k T_k) \\ p_{y,k} + T_k(v_k + a_k T_k)\sin(\phi_k + \omega_k T_k) \\ v_k + a_k T_k \\ a_k \\ \phi_k + \omega_k T_k \\ \omega_k \end{bmatrix} \tag{3}$$

The motion model can be used to predict the state at time $k+1$ based on the state estimate from time k. T_k is the time span between k and $k+1$. However, the model assumptions for prediction only hold to a certain extent. During the transition between the curved and straight parts of the running track, the assumption of a constant radius is violated, leading to mismodeling and errors in the filtered output. These errors can be reduced by including a priori knowledge about the motion of the target. It is known, that the athlete can only move along the predefined running track, therefore its movement is constrained. However, within the limits of the track, variations of the movements are possible. We propose to include this information by augmenting the motion model. In unconstrained Kalman Filters the a priori state estimate for time $k+1$ ($x_{k+1|k}$) is calculated with a motion model that takes the a posteriori state estimate of time k ($x_{k|k}$) as input. We suggest to augment the motion model with additional a priori knowledge of the tracked movement to improve the quality of the a priori state prediction and thus the filtered positions. This approach is visualized schematically in Fig. 2.

For the example of the athlete on the running track, the orientation and turning rate on the straight parts of the track are no longer predicted as in (3). Instead, the corresponding parts of the model change to $\phi_{k+1} = \phi_{track}(p_x, p_y)$ and $\omega_{k+1} = \omega_{track} = 0$.

Equivalently, for the curved parts of the track, the orientation is also predicted according to the track orientation $\phi_{track}(p_x, p_y)$ and the turning rate is modeled as $\omega_{k+1} = v_k / r_{track}(p_x, p_y)$. Hence, the turning rate depends on the current translational speed and the current radius of the track r_{track}.

The orientation of the track depends on the current position. It can be calculated as the orientation orthogonal to the curve normal. In the example of the running track, the curve can be analytically described as a semi-circle which gives

$$\phi_{track}(p_x, p_y) = arctan(\frac{p_y - p_{cy}}{p_x - p_{cx}}) + \frac{\pi}{2} \tag{4}$$

where (p_{cx}, p_{cy}) is the position of the curve center.

In this example, the a priori information that is used to augment the motion model changes according to the current position of the runner on the track and the corresponding track description. Elements of the state vector for which no a priori knowledge is available are predicted using the standard motion model described in (3). This allows a seamless change from an augmented to a generic motion model and vice versa.

Not only orientation and turning rate information, but also a priori knowledge about changing dynamics can be included in an augmented motion model. This can be achieved by changing from a constant acceleration to a constant velocity model. For example, in a running competition it can be assumed, that the speed of the runners is almost constant after the starting phase. Therefore, the model can be changed from a constant acceleration model to a constant velocity model.

Our approach is also applicable for more complex scenarios for which a closed-form description of the track is not available. In car navigation systems, the map representation of roads is usually piecewise linear, i.e. roads are represented as line segments [15]. To include such map information in the position tracking using our proposed method, the line segment that is closest to the filtered position needs to be determined. The orientation of this line segment can then be used to augment the motion model and to improve the quality of the filtered positions.

The proposed method does not require a change in the general structure of the Kalman filter, since only the motion model needs to be changed (see Fig. 2). Therefore, other Kalman filter modifications can be adapted to our method directly.

III. EXPERIMENTS AND RESULTS

A. Data Collection

In order to test our approach under realistic conditions, we collected 34 position data sets during 400 m and 800 m runs. The runs were performed by 8 different runners. In total, the runners covered a distance of approximately 18,000 m for the recordings. The data were collected on two different oval running tracks (400 m per round) whose geometries were compliant to the regulations of the International Association of Athletics Federations [14] (see Fig. 1).
The positions of the runners were measured in 2D using a LPS [16]. The LPS employed the time-difference-of-arrival (TDOA) principle to obtain the positions. The runners were equipped with two different LPS transponders. One transponder was designed to be small (approx. 6x4x0.5 cm) and lightweight in order to not disturb the runners. This transponder type will be referred to as measurement transponder. The other transponder was designed for optimal measurement accuracy and featured two external antennas to improve the measurement quality. This transponder type will be referred to as reference transponder. Due to their increased size and weight, the reference transponders were not suitable to be used in competitions. However, the measurement transponders suffered from weakened signal transmission between the transponder and the LPS base stations, which led to a reduced signal-to-noise ratio, outliers in the measurements and a variable sampling period with long outages of up to 13.7 s. In total, 82 258 position measurements were obtained. Position measurements from both transponders were recorded simultaneously during all runs.

B. Filter Comparison

1) Compared Filters: The trajectories obtained with the reference transponders were low-pass filtered with a cutoff frequency of 0.5 Hz to remove high-frequency noise. The low-pass-filtered filtered trajectories from the reference transponders served as the ground truth for the evaluation.

In our experiments, we applied six different Kalman-based filters to the data from the measurement transponders to obtain filtered positions for each runner. For comparison, the SRUKF and EKF in their basic forms (i.e. without constraints), and with the proposed augmented motion model described in section 3 were used. Since the most widely used method in the literature for the incorporation of soft constraints is the use of pseudo-measurements, the EKF and SRUKF with pseudo-measurements were also included in the comparison. Thus, the compared filters were the unconstrained EKF (U-EKF), the unconstrained SRUKF (U-SRUKF), the EKF with pseudo measurements (P-EKF), the SRUKF with pseudo measurements (P-SRUKF), the EKF with the augmented motion model (M-EKF) and the SRUKF with the augmented motion model (M-SRUKF). Each of these filters was applied to the position data from the measurement transponders.

The same soft constraints were used in the pseudo-measurement filters and the augmented motion model filters. These constraints included the assumptions that the orientation and turning rate of the runners could be approximated from the track geometry and that the acceleration of the athletes was zero after the starting phase. In combination with tested filtered variants we also applied a real-time capable outlier detection method, which was based on the Mahalanobis distance between the predicted measurement and the actual measurement [17].

2) Parameter Estimation: In order to obtain meaningful filter results, the optimal parameters for each of the tested Kalman-based filters were determined. The recorded data sets from the measurement transponders were heterogeneous regarding measurement quality and sampling rate. To account for these variations, four different data sets were used for estimation of the filter parameters: the data sets with the highest and lowest average sampling rate and the data sets with the highest and lowest root mean square error (RMSE) of the measurements. These data sets were only used for the estimation of the filter parameters. They were excluded from the evaluation of the filters.

The measurement noise covariance matrix was determined for each filter as described in [18]. To avoid distortion of the measurement noise estimate, outliers were identified with a Grubbs test [19] and were not included in the estimation of the measurement noise. The optimal parameters for the process noise covariance matrix, the measurement noise for the pseudo measurements and the Mahalanobis distance threshold for detecting outliers in real-time were determined for each filter with a genetic algorithm for global optimization [17]. Note that all measurements *including* outliers were used for the evaluation of the different filters with the remaining 30 data sets.

3) Evaluation: The output of the different filters was evaluated regarding the accuracy of the filtered positions, robustness towards measurement errors, robustness towards measurement outages and calculation time.

For the evaluation of the accuracy, the root mean square error (RMSE) between the filtered data from the measurement transponders and the ground truth from the reference transponders was calculated. RMSE values were obtained separately for each tested data set and each tested filter. For each of the filters, the mean and standard deviation of the RMSE values over the 30 tested data sets were obtained.

To gain further insight into the influencing factors of the filtered output, the relation between the RMSE of the measurements and the RMSE of the filtered output was analyzed. To this end, a linear regression model was fitted for each filter output. The model was based on the results from all test sets.

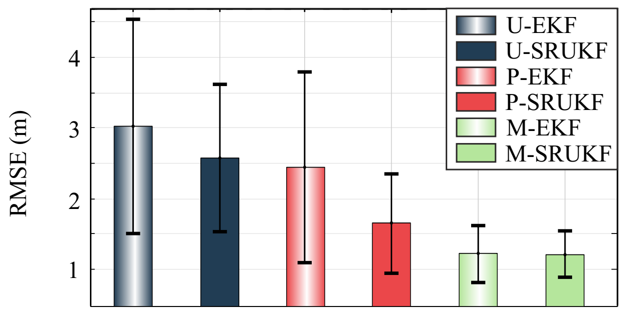

Figure 3. RMSE of the filtered positions from the different filters. The bars corresponds to the mean RMSE over all test sets, the standard deviation is indicated by the vertical lines.

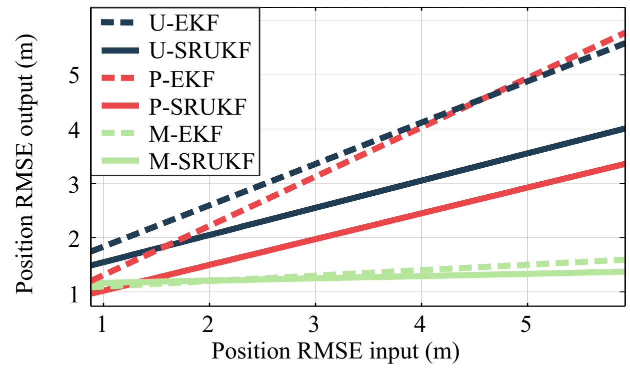

Figure 4. Relation between RMSE of measured input and RMSE of filtered output. Dashed lines correspond to the regression models of the EKFs, solid lines correspond to the regression models of the SRUKFs.

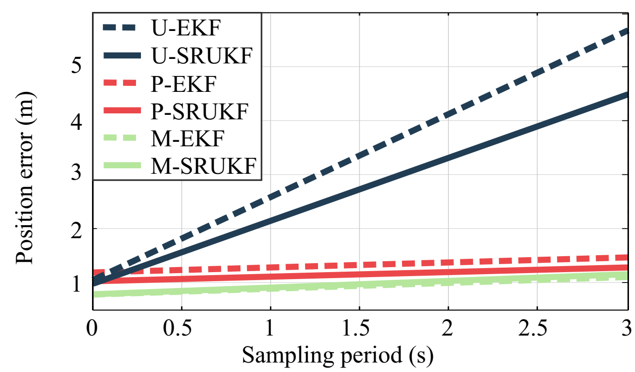

Figure 5. Relation between sampling period of the measurements and error of the filtered positions. Dashed lines correspond to the regression models of the EKFs, the solid lines correspond to the regression models of the SRUKFs.

Moreover, the position error was set in relation to the sampling period of the measurement system. For this purpose, a linear regression model was calculated, based on the deviation of each filtered position from the reference and the time since the last measured input was received.

In addition, the calculation time was determined for each filter. Since the absolute calculation times depend on the used hardware, we determined relative calculation times. All calculation times were determined in relation to the time that was required for the unconstrained EKF, which served as the baseline.

C. Results

The mean and standard deviation of the RMSE of the filtered positions are depicted in Fig. 3. The filter output of the U-EKF had the highest average RMSE (3.01 m). The lowest errors were obtained with the M-EKF and M-SRUKF (both 1.21 m). The models relating the quality of the measured input to the filtered output are shown in Fig. 4. The models relating the sampling rate of the input to the quality of the filtered output result are visualized in Fig. 5.

Compared to the unconstrained EKF, the calculation time for the EKF with the proposed augmented motion model increased by 19 % and by 108 % for the P-EKF. Compared to the unconstrained SRUKF, the increase of calculation time was 58 % for the M-SRUKF and 127 % for the P-SRUKF. The comparison of calculation times is visualized in Fig. 6.

IV. DISCUSSION

The comparison between the different filtering approaches shows that the SRUKF with the proposed augmented motion model exhibited the best performance in terms of the RMSE of the filtered output (Fig. 3). The EKF in combination with the augmented motion model yielded almost the same results. This conformity can be attributed to the fact, that in our test data the change in movement dynamics was relatively slow. In these cases, the linearization errors of the EKF are negligible and the use of an unscented filter is not necessary [20].

Including pseudo-measurements improved the results compared to the unconstrained filters. The RMSE was further reduced in the cases where the augmented motion model was used. The differences in the RMSE of the measurements were mainly caused by differing occurrences of outliers. Therefore, in our case, robustness towards outliers means that the influence of the input RMSE on the output RMSE should be small. The small slopes of the fitted models (Fig. 4) of the M-SRUKF and M-EKF indicate, that these filters are robust towards outliers. In contrast, the pseudo-measurement filters were much more susceptible to low measurement quality caused by outliers. This can be explained by the influence of the pseudo-measurements on the state covariance. Julier and LaViola [21] argue, that the unconstrained estimate has minimum variance and hence, the application of a constraint should lead to an *increase* of the state covariance. However, the application of the pseudo-measurements leads to a *decrease* of the covariance. Since the outlier detection was based on the Mahalanobis distance which in turn is a function of the state covariance, an implausible covariance matrix led to a deteriorated outlier detection. The high susceptibility of the unconstrained filters can be explained by the fact that both outliers and inaccurate state predictions due to an insufficient motion model led to a high Mahalanobis distance and thus the two effects could not be distinguished.

The advantage of using constraints for position tracking

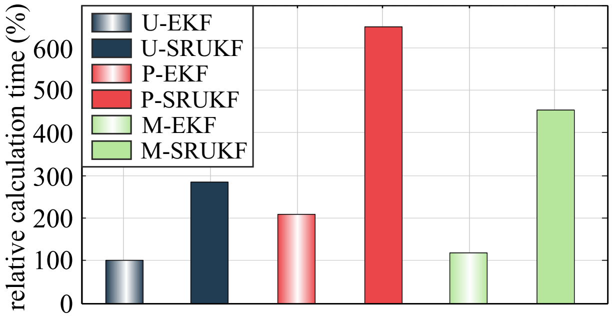

Figure 6. Relative calculation times for the compared filters. The calculation time for the unconstrained EKF is defined as 100 %.

is emphasized by the results shown in Fig. 5. Even when measurement outages occurred, the quality of the filtered output in our tests deteriorated only slightly if constraints were applied.

The evaluation of the calculation times of the different filters shows that the use of the proposed augmented motion model led to only small changes in the calculation time when compared to the unconstrained filters. In contrast, including pseudo-measurements led to a considerable overhead. The pseudo-measurements increase the dimensionality of the measurement vector, thus increasing the computational cost of the update step. The exact quantitative results for the calculation times depend on the implementation and application. Therefore, our evaluation of the calculation times should be treated only as a qualitative assessment. Regarding computational cost, it should also be considered, that the use of pseudo-measurements requires more filter parameters (the pseudo-measurement noise) to be estimated, which increases the computational cost of the filter design. In contrast, the filters with the proposed augmented motion models do not require additional parameters compared to the unconstrained model as described in (3).
A drawback of the constrained filters is the loss of generality, i.e. the constraints need to be adapted and remodelled for each specific application. The use of the proposed augmented motion model also requires the adaption of the Jacobian of the motion model in the M-EKF. If used in conjunction with a SRUKF or UKF the augmented model needs to be evaluated separately for each of the sigma points, which increases the computational cost compared to an unconstrained filter.

V. CONCLUSION

In this paper, we introduced the concept of augmented motion models to incorporate soft constraints in Kalman filters. Our method does not require additional parameters or a change in the general structures of the filter, which facilitates implementation. We showed that our method outperformed unconstrained filters and pseudo-measurement filters in a realistic scenario. Of the analyzed methods, our approach yields the best results regarding overall accuracy of the filtered data, robustness towards outliers and robustness towards measurement outages. At the same time, it requires only small computational overhead in comparison to an unconstrained solution. The application of the proposed method is not limited to sports but can also augment the tracking in various other scenarios where it allows optimal use of a priori knowledge.

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